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## Liquid Crystalline Compensator

Fedor V. Podgornov<sup>a</sup>, Victor A. Krivoschokov<sup>a</sup> & Igor B. Tsarev<sup>a</sup>

<sup>a</sup> Nonlinear Optics Laboratory, Technical University, Lenin ave., 76, Chelyabinsk, 454080, Russia

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## **Liquid crystalline compensator**

FEDOR V. PODGORNOV, VICTOR A. KRIVOSCHOKOV,  
IGOR B. TSAREV  
Nonlinear Optics Laboratory, Technical University,  
Lenin ave., 76, Chelyabinsk 454080, Russia

A device for measuring of the parameters of the state of optical polarization has been proposed and experimentally realized. It has been demonstrated that every state of polarization has unique intensity distribution behind the proposed device. This compensator allows us to measure the required parameters of the polarization state in the real-time regime.

**Keywords:** liquid crystal; compensator; optical polarization

## **INTRODUCTION**

Measuring of the parameters of the state of light polarization is a very important problem. Usually, various types of automatic ellipsometers that are based on rotating analyzers<sup>[1]</sup>, rotating compensators<sup>[2]</sup>, photoelastic cells<sup>[3]</sup>, and electro-optical crystals<sup>[4]</sup> are used for this purpose. However, such devices are complex and expensive. In addition to that, they contain mechanical parts, hence they require fine tuning. Therefore, it is very difficult to use them for measuring parameters of the state of polarization in real-time regime. In work<sup>[5]</sup>, a new device has been proposed which can be used for real-time measurements. The intensity distribution behind it depends on the state of polarization of the incident light beam. It is very important to notice

that the form of intensity distribution does not depend on the wavelength. However, these patterns are not trivial. Therefore, in order to analyze them and derive necessary information about the polarization state, special software program and equipment are required. In this paper, we propose new a device which is simpler and more effective than above mentioned ones.

## THEORY

The proposed device consists of a wedge-form cell and a linear polarizer. The cell is filled with a nematic liquid crystal. The liquid crystal is supposed to have homogeneous orientation.

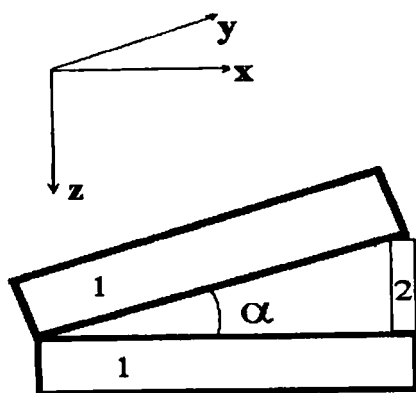


FIGURE 1 Scheme of compensator

The angle between the director of the liquid crystal and the transmission axis of a polarizer is  $\alpha$ . In this case, the thickness of a liquid crystalline layer in the point with coordinate  $x$  can be written as follows:

$$l(x) = x \cdot \tan(\varphi) \quad (1)$$

Now, we consider an elliptically polarized plane wave of wavelength  $\lambda$  impinging at normal incidence along  $Z$ -axis onto the device. For simplicity

sake, we will neglect of reflection from the cell and absorption in the liquid crystal. The further investigation of process of propagation of polarized light through the compensator will be based on Jones matrix formalism<sup>[6]</sup>. The inner angle of a wedge-form cell is  $\varphi$ .

Suppose, that the impinging monochromatic plane wave  $\vec{E} = E_0 \cdot e^{kr - \Omega t}$  has an arbitrary polarization, that is, in general case, elliptical polarization. So, we can write its Jones vector as:

$$E = \begin{pmatrix} a \\ b \end{pmatrix} \quad (2)$$

where  $a = E_x / \sqrt{I}$ ,  $b = E_y / \sqrt{I}$ ,  $I$  - intensity of the analyzed beam,  $E_x$  and  $E_y$  are components of strength of electric field along **X** and **Y** axes, respectively. According to Jones matrix formalism, the Jones vector of transmitted light beam can be written in the following form:

$$E' = R(-\alpha) \cdot P \cdot R(-\alpha) T(x) \cdot E \quad (3)$$

Here are:

$$R(\alpha) = \begin{pmatrix} \cos(\alpha) & \sin(\alpha) \\ -\sin(\alpha) & \cos(\alpha) \end{pmatrix} \quad (4)$$

-the rotation matrix of the coordinate system at angle  $\alpha$  around **Z** axis,

$$R(\alpha) = \begin{pmatrix} \cos(\alpha) & -\sin(\alpha) \\ \sin(\alpha) & \cos(\alpha) \end{pmatrix} \quad (5)$$

-matrix of the inverse rotation,

$$T(x) = \begin{pmatrix} e^{-1/2 \Delta n l(x) k i} & 0 \\ 0 & e^{1/2 \Delta n l(x) k i} \end{pmatrix} \quad (6)$$

- the transmission matrix of a liquid crystal layer at point with coordinate **X**,

$$P = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \quad (7)$$

-Jones matrix of the polarizer with transmission axis oriented along **Y**-axis.

Substituting Eqs.(2), (4), (5), (6), and (7) into Eq.(3) one can obtain the expressions for the components of the Jones vector of transmitted light beam in the following form:

$$a' = a \sin^2(\varphi) e^{-i\Gamma/2} - 1/2 b \sin(2\varphi) e^{i\Gamma/2} \quad (8)$$

$$b' = -1/2 a \sin(2\varphi) e^{-i\Gamma/2} + b \cos^2(\varphi) e^{i\Gamma/2} \quad (9)$$

where  $\Gamma = \Delta n l(r) k$ . Using formulas (8) and (9), one can get the normalized intensity distribution of transmitted beam in the most general case:

$$I = |a|^2 \sin^2(\varphi) + |b|^2 \cos^2(\varphi) - \sin(2\varphi) \operatorname{Re}(a \cdot b^* \cdot e^{-i\Gamma}) \quad (10)$$

As it is apparent from this formula that every state of polarization of incident light beam has its own, unique intensity distribution behind the device. Now, let us obtain formulas of intensity distribution for some, widely used, states of optical polarization: linear vertical polarization (y- polarization), linear polarization with azimuth  $45^\circ$ , linear polarization with azimuth  $135^\circ$ , linear horizontal polarization (x-polarization), left-handed circular polarization, right-handed circular polarization. The results of calculations are summarized in Table 1.

In order to derive all information about the state of polarization, it is necessary that phase difference between waves with orthogonal polarization take values from 0 to, at least,  $2\pi$  along the **X**-axis. This requirement can be written as:

$$\Delta n k l(x) = 2\pi$$

where  $l(x) = x \cdot \tan(\varphi)$ .

TABLE 1 Formulas of intensity distribution for some states of light polarization

Jones vector	Formula of intensity distribution
$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$	$\cos^2(\varphi)$
$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$	$\sin^2(\varphi)$
$\begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix}$	$\frac{1}{2} - \frac{1}{2}\cos(\Gamma)\sin(2\varphi)$
$\begin{pmatrix} -1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix}$	$\frac{1}{2} + \frac{1}{2}\cos(\Gamma)\sin(2\varphi)$
$\begin{pmatrix} 1/\sqrt{2} \\ i/\sqrt{2} \end{pmatrix}$	$\frac{1}{2} + \frac{1}{2}\sin(\Gamma)\sin(2\varphi)$
$\begin{pmatrix} 1/\sqrt{2} \\ -i/\sqrt{2} \end{pmatrix}$	$\frac{1}{2} - \frac{1}{2}\sin(\Gamma)\sin(2\varphi)$

So, the minimal length of the illuminated spot should be:

$$x = \frac{2\pi}{\Delta n k \tan(\varphi)}$$

Typical graphics of intensity distribution are shown in fig 3. For their calculation we used the parameters  $\Delta n=0.2$ ,  $\lambda=0.6328 \mu\text{m}$ ,  $\varphi=7 \cdot 10^{-4}$  rad, and the following states of polarization: linear polarization with azimuth  $45^\circ$  (1), linear polarization with azimuth  $135^\circ$  (2), left-handed circular polarization (3), right-handed circular polarization (4). It is apparent from these graphics that every state of polarization has its unique intensity distribution behind the device.

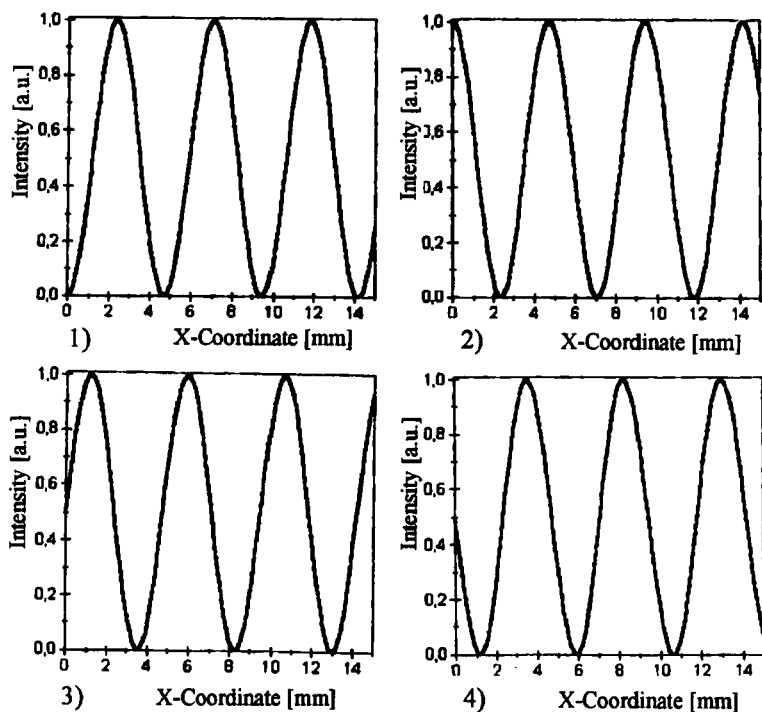


FIGURE 2 Graphics of intensity distributions (computer simulation)

## EXPERIMENT

The compensator was made from two quartz plates (1) (see fig.1). Each plate is 30 mm length and 25 mm width. The planar orientation of the nematic was achieved by rubbing of the quartz plates in the direction of the short side with diamond paste. The Teflon separator (2) with thickness  $20\text{ }\mu\text{m}$  was placed at the one end of the cell in such a way that it was parallel to the rubbing direction. So, one plate was tilted with respect to the other at angle  $\varphi=7\cdot 10^{-4}$  rad. The space between the two plates was filled with NLC 5CB. The quality



of the crystal orientation was controlled by placing the cell between crossed polarizers. The experimental setup for investigation of the proposed device is shown in fig. 3.

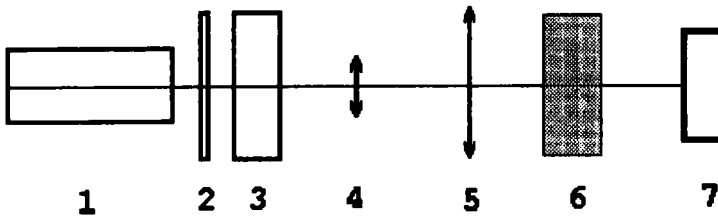


FIGURE 3 Experimental setup

A He-Ne laser (1), operating at  $\lambda=0.6328 \mu\text{m}$ , was used to provide the light beam. For getting light with any desired state of polarization, tunable  $\lambda/4$  plate (2,3) was used<sup>[7]</sup>. It provided quality of polarization  $0.99 \pm 0.01$ . The beam expander consisting of two lenses (4,5) provided the required transverse size of the beam. Behind the expander the compensator (6) was placed. In our experiment, the angle between the transmission axis of the polarizer and the director was  $45^\circ$ . Registration of the intensity distribution was implemented either visually on the screen (7) or by means of a CCD matrix placed instead of screen. In this experiment, we selected the following states of light polarization: linear vertical polarization (y -polarization), linear polarization with azimuth  $45^\circ$  (1), linear polarization with azimuth  $135^\circ$  (2), linear horizontal polarization (x- polarization), left-handed circular polarization (3), right-handed circular polarization (4). The graphics of experimentally obtained intensity distributions are shown in fig.(4). From this figure, it is apparent that every state of polarization has its own intensity distribution behind the compensator.

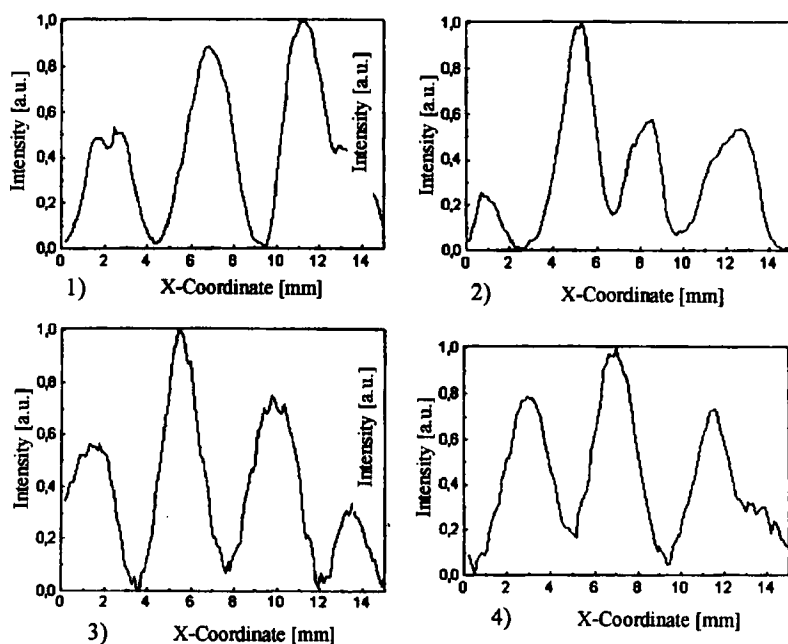


FIGURE 4 Intensity distributions behind the compensator (experimental results)

## CONCLUSION

In this work, we have proposed and experimentally realized a device which allows us to measure the parameters of the polarization state in real-time regime. This device is a modification of the device (polariscope) realized earlier. But our compensator has simpler construction and can be easily conjugated with a computer. Moreover, the patterns of intensity distribution are very easy to analyze.

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